Rapid Mixing via Coupling Independence for Spin Systems with Unbounded Degree

Xiaoyu Chen
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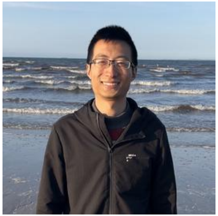
based on joint work with



Weiming Feng
The University of Hong Kong

Sampling from spin systems

q-spin systems

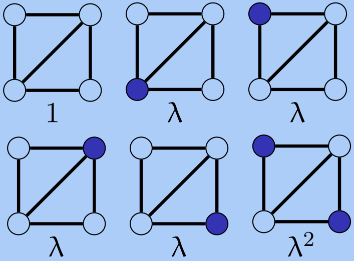
Fix $q \geqslant 2$. Let $G = (V, E)$ be a graph and

- let $A \in \mathbb{R}_{\geqslant 0}^{q \times q}$ be the interaction matrix
- $\forall v \in V$, let $\mathbf{b}_v \in \mathbb{R}_{\geqslant 0}^{q}$ be external fields

Gibbs distribution μ: $\forall \sigma \in [q]^V$,

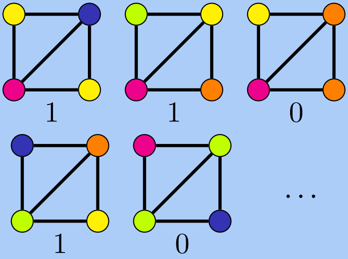
$$\mu(\sigma) \propto \prod_{e=\{u,v\} \in E} A(\sigma_u, \sigma_v) \prod_{v \in V} \mathbf{b}_v(\sigma_v)$$

hardcore model (Ind. set)



$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{b}_v = \begin{pmatrix} 1 \\ \lambda \end{pmatrix}$$

uniform (proper) q-coloring



$$A = \mathbf{1}_q^{\mathsf{T}} \mathbf{1}_q - \mathbb{I}_q, \quad \mathbf{b}_v = \mathbf{1}_q$$

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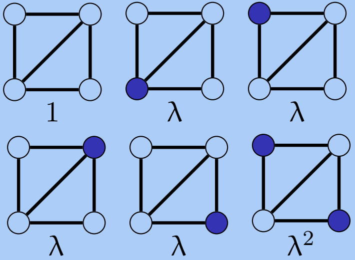
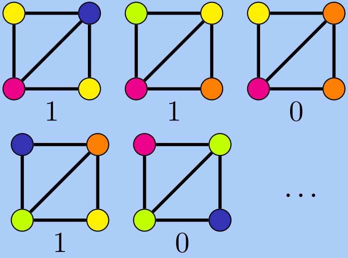


$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{b}_v = \begin{pmatrix} 1 \\ \lambda \end{pmatrix}$$

uniform (proper) list-coloring



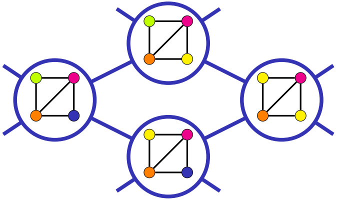
$$A = \mathbf{1}_q^{\intercal} \mathbf{1}_q - \mathbb{I}_q, \quad \mathbf{b}_v \in \{0, 1\}^q$$

MCMC method

Glauber dynamics is the **standard** algorithm to sample from Gibbs distributions

it updates the current state X as

1. pick a vertex u uniformly at random
2. resample X_u from $\mu_u(\cdot \mid X_{V \setminus u})$



$$X_0 \to X_1 \to X_2 \to X_3 \to \cdots \to X_t$$

MCMC method

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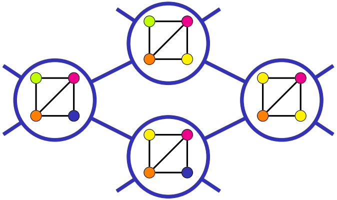
1. pick a vertex u uniformly at random
2. let X_u be a new random color that does not appear in u's neighbors



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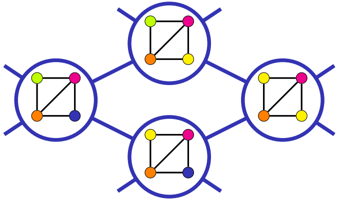

$$X_0 \to X_1 \to X_2 \to X_3 \to \cdots \to X_t$$

MCMC method: run GD. for t steps (with sufficiently large t); then output X_t

- **Convergence**: $X_t \sim \mu$ as $t \to \infty$
- **Mixing time**: $T_{\mathsf{mix}}(\mu) := \max_{S_0} \min \{t \mid d_{TV}(X_t, \mu) \leqslant 1/4\}$
- **TV-distance**: $d_{TV}(X_t, \mu) := \sum_s |\mathbf{Pr}[X_t = s] - \mu(s)|$
- **Relaxation time**: $T_{\mathsf{rel}}(\mu) := \frac{1}{1-\lambda_2}$ λ_2: the 2nd largest eigenvalue of GD.

$$T_{\mathsf{mix}}(\mu) \leqslant T_{\mathsf{rel}}(\mu) \log \frac{4}{\min_\sigma \mu(\sigma)}$$

The **bounded degree** requirement
in the mixing time analysis

The **bounded degree** requirement in the mixing time analysis

spectral independence $\Rightarrow$ rapid mixing [ALO20, CLV20, CGŠV21, FGYZ21]:

- influence matrix $\Psi(\mu) \in \mathbb{R}^{qn \times qn}$: let $X \sim \mu$,

$$\Psi(\mu)_{ia,jb} := \begin{cases} \mathbf{Pr}\left[X_j = b \mid X_i = a\right] - \mathbf{Pr}\left[X_j = b\right], & \mathbf{Pr}\left[X_i = a\right] > 0 \\ 0, & \mathbf{Pr}\left[X_i = a\right] = 0 \end{cases}$$

- $\mathsf{SI}(\mu) = \max_\tau \lambda_{\max}(\Psi(\mu^\tau))$, where the maximum enumerates all feasible partial configurations τ.
- consequence: $T_{\mathsf{mix}}(\mu) \approx T_{\mathsf{rel}}(\mu) = n^{O(\mathsf{SI}(\mu))}$.

for many important spin systems, SI is proven to be a universal constant

- anti-ferro. two-spin systems in uniqueness regime [ALO20, CLV20]
- proper q-coloring on triangle-free graph with $q \geqslant 1.763\Delta$ [CGŠV21, FGYZ21]
- even-subgraph model with penalty $\eta > 0$ [CZ23]
- bipartite hardcore model with “one-sided” uniqueness [CLY23]
- $\ldots$

The **bounded degree** requirement in the mixing time analysis

SI $\Rightarrow$ optimal mixing (assuming bounded max degree Δ) [CLV21]

$$T_{\text{mix}}(\mu) = \Delta^{\Delta^{O(\text{SI}(\mu))}} n \log n$$

$$T_{\text{rel}}(\mu) = \Delta^{O(\text{SI}(\mu))} n$$

Question: Can we get optimal bounds for arbitray graph?

$$T_{\text{mix}} = O(n \log n) \quad \text{and} \quad T_{\text{rel}} = O(n)$$

Now, we have an affirmative answer for two-spin systems ($q = 2$)

improve Δ to $O(1)$

- field dynamics [CFYZ21]
- localization scheme [CE22]

improve Δ to $O(1)$ [AJKPV22]

- entropic independence ($\text{EI}(\mu)$)
  $\text{EI}(\mu) = O(1) \quad \Rightarrow \quad \text{SI}(\mu) = O(1)$

- EI for Ising model with $\|J\|_2 < 1$ [AJKPV22]
- EI for general anti-ferro. 2-spin system in uniqueness regime [CE22, CFYZ22]
- ...

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generalization for $q > 2$?

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analysis EI when $q > 2$?

Question: Optimal T_{rel} and T_{mix} for multi-spin systems ($q > 2$)?

Our results

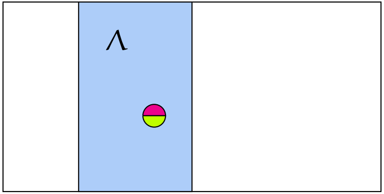
Our results

coupling independence

$$\mathsf{CI}(\mu) := \max_{\substack{\Lambda \subseteq [n] \\ \sigma,\tau \in [q]^{\Lambda}, |\sigma \oplus \tau| = 1}} \mathcal{W}_1(\mu^{\sigma}, \mu^{\tau})$$

W1-distance

$\mathcal{W}_1(\mu^{\sigma}, \mu^{\tau}) := \inf_{\mathcal{C}} \mathbb{E}_{(X,Y)\sim\mathcal{C}} [|X \oplus Y|]$,
$\mathcal{C}$: coupling between μ^{σ} and μ^{τ}



$\mathsf{CI}(\mu) = O(1)$ means for any two partial configurations σ, τ that only differ at one vertex, there is a coupling $\mathcal{C}$ between μ^{σ} and μ^{τ} such that the expected difference is at most constant

- $\mathsf{CI}(\mu) = O(1) \Rightarrow \mathsf{SI}(\mu) = O(1)$ [CZ23]
- CI has been used implicitly or explicitly in many previous works [Liu21, BCCPŠV21, CZ23, CG24, CLMM23, Jer24]

Our results

μ: the uniform distribution of proper q-coloring on a graph G with maximum degree Δ

Theorem

For $\varepsilon \in (0, 1)$, if $q \geqslant (1 + \varepsilon)\Delta$, then

- there is a fast sampler for μ that runs in time $\Delta n(\log n)^{\mathrm{poly}(\varepsilon^{-1}, \mathrm{CI}(\mu))}$
- we have a bound for $T_{\mathrm{rel}}(\mu) = e^{\mathrm{poly}(\varepsilon^{-1}, \mathrm{CI}(\mu))} n$

When the graph is triangle-free and $q/\Delta > 1.763$, then $\mathrm{CI}(\mu) = O(1)$ [FGYZ21]

Corollary

If G is triangle-free and $q/\Delta > 1.763$, then

- there is a fast sampler for μ that runs in time $\Delta \widetilde{O}(n)$
- we have a bound for $T_{\mathrm{rel}}(\mu) = O(n)$

Compare to previous result for the same setting in [JPV21]:

- $T_{\mathrm{rel}}(\mu) = n^{1+o(1)}$
- $\Rightarrow$ a sampler that runs in time $\Delta n^{2+o(1)}$

Our results

Our technique also works for two-spin systems

Lemma informal

For two-spin system, $\mathsf{CI}(\mu) \leqslant$ total influence on a self-avoiding walk tree

$\Rightarrow \mathsf{CI} = O(1)$ for hardcore / Ising model in the uniqueness regime

Recover the known results for hardcore / Ising model in the uniqueness regime

- $T_{\mathsf{rel}}(\mu) = O(n)$
- a fast sampler for μ that runs in time $\Delta \widetilde{O}(n)$

Our results

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$\Rightarrow \mathsf{CI} = O(1)$ for hardcore / Ising model in the uniqueness regime

Let $G = (V_L \uplus V_R, E)$ be a bipartite graph

- let Δ_L (Δ_R resp.) denote the maximum degree for vertices in V_L (V_R resp.)
- the uniqueness threshold for the fugacity λ: $\lambda_c(\Delta) := \frac{(\Delta-1)^{(\Delta-1)}}{(\Delta-2)^{\Delta}}$

Let μ be the hardcore model with fugaicity λ on G

Theorem

For $\delta \in (0,1)$, $C > 0$, if $\lambda \leqslant (1-\delta)\lambda_c(\Delta_L)$ and $\Delta_R = C \times \Delta_L$, then

- there is a fast sampler for μ that runs in time $\Delta\widetilde{O}(n)$
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Proof overview

μ: the uniform distribution of proper q-coloring on a graph G with maximum degree Δ

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μ: the uniform distribution of proper list-coloring on a graph G with maximum degree Δ

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For $\varepsilon \in (0, 1)$, if $\forall v$, $\|\mathbf{b}_v\|_1 \geqslant (1 + \varepsilon)\Delta$, then

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Recall: $\mathbf{b}_v$ is a 01-vector indicating the avaliable colors for vertex v

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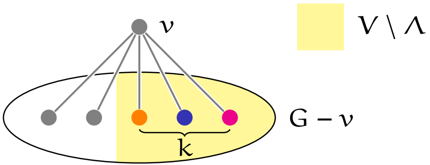
Observation: self-reducibility & "easier" conditional distributions

1. Let $\Lambda \subseteq V$, for $\tau \in [q]^{V \setminus \Lambda}$, μ^τ_Λ is a uniform distribution of list-coloring on $G[\Lambda]$ with new color list $\mathbf{b}'_v$ for each vertex $v \in \Lambda$



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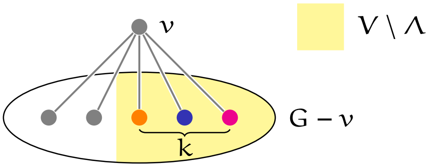

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2. If $\frac{\|b'_v\|_1}{\Delta'_v} > 5$, for all $v \in \Lambda$, we know $T_{\mathrm{mix}}(\mu_\Lambda^\tau) = O(n \log n)$ [DB97]

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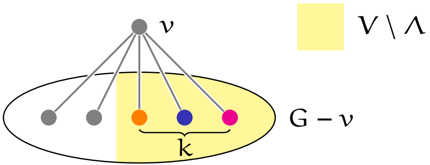

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3. Suppose that $G[\Lambda]$ has maximum degree $\Delta' \leqslant \theta\Delta$ for some $\theta \in (0,1)$

$$\frac{\|\mathbf{b}'_v\|_1}{\Delta'_v} \geqslant \frac{\|\mathbf{b}_v\|_1 - k}{\Delta'_v = (\Delta_v - k)} \geqslant \frac{\varepsilon\Delta + (\Delta - k)}{\Delta'_v = (\Delta_v - k)} \geqslant 1 + \frac{\varepsilon}{\theta}$$

Reduce the sampling task to "easier" conditional distributions

for $\theta \in [0, 1]$, let $\mathsf{EZ}(\mu, \theta)$ be the family of "easier" conditional distributions of μ

$$\mathsf{EZ}(\mu, \theta) := \left\{ \mu_\Lambda^\tau \;\middle|\; \begin{array}{c} \Lambda \subseteq V \text{ s.t. max degree of } G[\Lambda] \leqslant \theta\Delta \\ \tau \in [q]^{V \setminus \Lambda} \end{array} \right\}$$

worst case mixing / relaxation time for "easier" conditional distributions

$$T_{\mathsf{rel}}^{(\theta)}(\mu) := \max \{ T_{\mathsf{rel}}(\nu) \mid \nu \in \mathsf{EZ}(\mu, \theta) \}$$

$$T_{\mathsf{mix}}^{(\theta)}(\mu) := \max \{ T_{\mathsf{mix}}(\nu) \mid \nu \in \mathsf{EZ}(\mu, \theta) \}$$

Theorem

For any $\theta \in (0, \frac{1}{2\mathsf{CI}(\mu)})$, there exists $\Delta_0(\theta, \mathsf{CI}(\mu))$ such that when $\Delta \geqslant \Delta_0$, then

- $T_{\mathsf{rel}}(\mu) = 2^{O(\mathsf{CI}(\mu)/\theta)} \times T_{\mathsf{rel}}^{(\theta)}(\mu)$
- there is a fast sampler for μ that runs in time $\Delta(\log n)^{O(\mathsf{CI}(\mu)/\theta)} \times T_{\mathsf{mix}}^{(\theta)}(\mu)$

- When $\Delta \leqslant \Delta_0$, we have $T_{\mathsf{mix}}(\mu) = O(n \log n)$ directly from [CLV21]
- In the rest part of this talk, we will focus on the second bullet

A Russian doll approach to sample from μ

k-partition

$V = U_1 \uplus U_2 \uplus \cdots \uplus U_k$

For $\Lambda \subseteq [k]$, let $U_\Lambda := \cup_{i \in \Lambda} U_i$

A Russian doll approach to sample from μ

block dynamics for μ

1. select $i \in [k]$ u.a.r.
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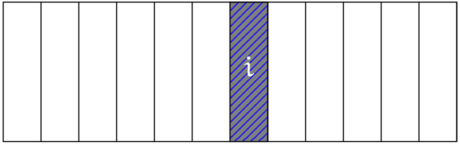

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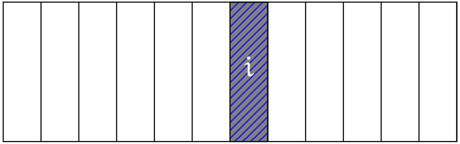

block dynamics of $\mu(\cdot \mid X_{U_i})$

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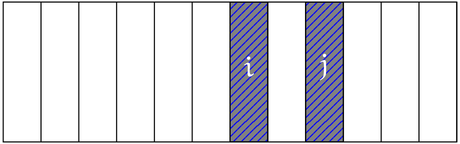

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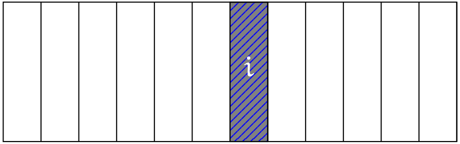

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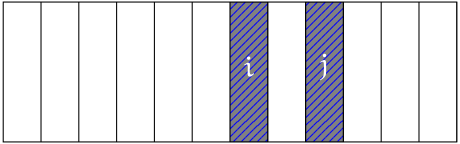
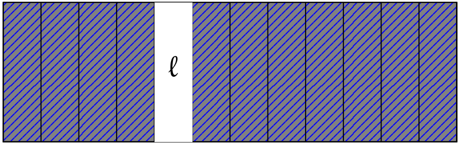


$\cdots$

primitive

sample from $\mu(\cdot \mid X_{U_{[k]\setminus \ell}})$

$\Leftrightarrow$ sample from $\mu_{U_\ell}(\cdot \mid X_{V \setminus U_\ell})$



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In order to get a fast sampler:

1. block dynamics fast mixing [next slide]
2. implement the primitives

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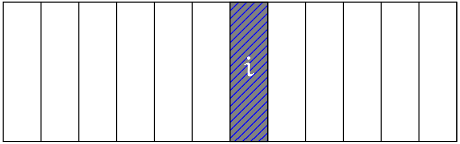

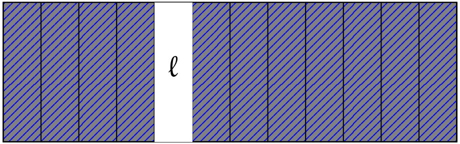

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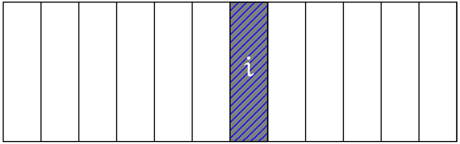
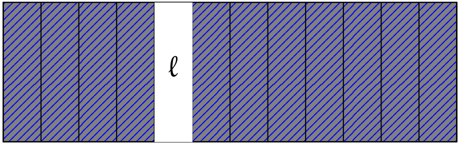


By using algorithmic LLL, we can

- construct a good partition s.t.
  $\forall X, \forall \ell, \quad \mu_{U_\ell}(\cdot \mid X_{V\setminus U_\ell}) \in EZ(\mu, \theta)$



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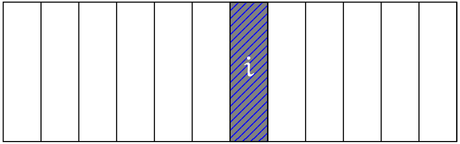
For $\Lambda \subseteq [k]$, let $U_\Lambda := \cup_{i \in \Lambda} U_i$

block dynamics for μ

1. select $i \in [k]$ u.a.r.
2. resample $X \sim \mu(\cdot \mid X_{U_i})$



In order to get a fast sampler:

1. block dynamics fast mixing [next slide]
2. implement the primitives

By using algorithmic LLL, we can

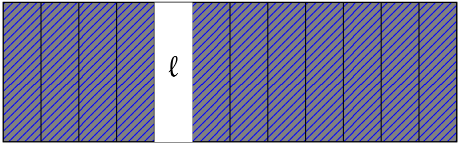
- construct a good partition s.t.
  $\forall X, \forall \ell, \quad \mu_{U_\ell}(\cdot \mid X_{V \setminus U_\ell}) \in EZ(\mu, \theta)$

intuition: distribute neighbors evenly into each partition

primitive

sample from $\mu(\cdot \mid X_{U_{[k]\setminus \ell}})$

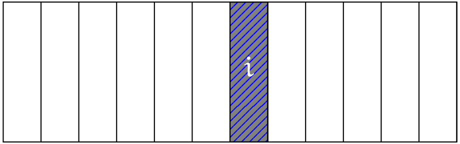
$\Leftrightarrow$ sample from $\mu_{U_\ell}(\cdot \mid X_{V \setminus U_\ell})$



Fast mixing of block dynamics

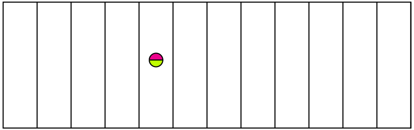
block dynamics for μ

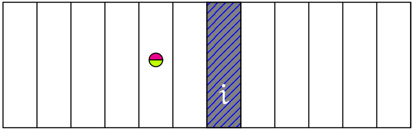
1. select $i \in [k]$ u.a.r.
2. resample $X \sim \mu(\cdot \mid X_{U_i})$



Path coupling

for $x, y \in [q]^V$ s.t. $|x \oplus y| = 1$, we have $\mathsf{CI}(\mu) < k \implies \mathcal{W}_1(\delta_x P_B, \delta_y P_B) < 1$





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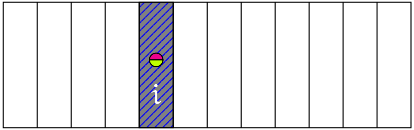

$\mathsf{CI}(\mu)/k$

Thank you

arXiv:2407.04672

Some interesting questions:

Compare our algorithm to Glauber dynamics?

Our algorithm $\approx$ censored Glauber dynamics

- monotone spin systems ✅
- general spin systems ❓

This is a new way to show $T_{\mathrm{mix}}(\mu) = \widetilde{O}(n)$

Better CI bound for coloring?

Many previous works only show $\mathrm{CI}(\mu) \leqslant \mathrm{poly}(\Delta)$

- $\mathrm{CI}(\mu) = O(1)$ when $q \geqslant (1+\varepsilon)\Delta$ ❓